Problems on Probability

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1. You have a hat in which there are three pancakes: one is golden on both sides, one is brown on both sides, and one is golden on one side and brown on the other. You withdraw one pancake, look at one side, and see that it is brown. What is the probability that the other side is also brown?

There are 3 pancakes to choose from. Only 2 of these 3 have at least one brown side. The pancake you picked has one brown side, so it must be one of the 2 pancakes from the above statement. You can think of the brown side on the brown and gold pancake as b_1 and the 2 brown sides on the brown and brown pancake as b_2 and b_3. If the side you are looking at is b_1, the other side is gold. If it is b_2 or b_3, the other side is brown also. Thus the probability of the other side also being brown is 2/3.

2. Three cities, A, B, and C, are located in a straight line, with B in the middle. A guy lives in B, his mom lives in A, and his girlfriend lives in C. Both the mom and the girlfriend insist on meeting the guy frequently. There is a train from B to A every 10 minutes and a train from B to C every 10 minutes. In order to be fair, every weekend/holiday, the guy arrives at the station at a random time, finds the first available train to A or C, and boards it, thus randomly deciding between his mom and his girlfriend. After a year of this arrangement, his girlfriend complains that he visits her only 20% of the weekends, and visits his mom 80% of the weekends. Is she simply being possessive, or could this actually be true?

This could actually be true if the train to the mom's house left at 9:08 and the train to the girlfriend's house left at 9:10 (or any two times that are separated by 2 minutes, with the mom first). This is because there is

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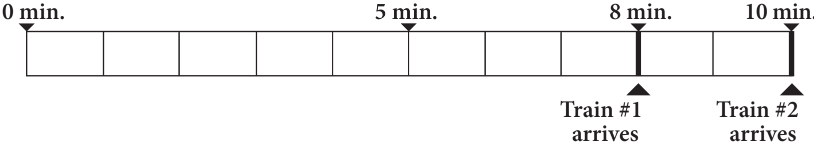
an 8-minute window in which the guy will be forced to go to the mom's house, but only a 2-minute window in which he is forced to go to the girlfriend's house. Think about it logically:

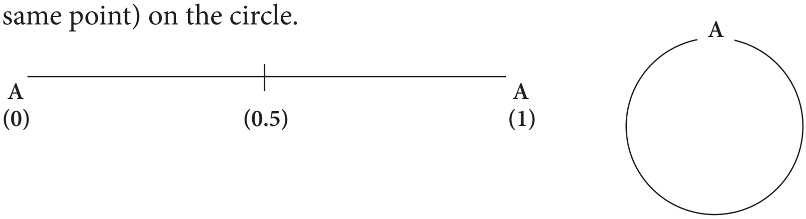
<u>Timeline</u>



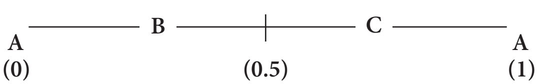
If he arrives at any time between the 1st and 2nd bar (0-8 minutes), he will take the train to his mom's house. If he arrives anywhere between the 2nd bar and the end (8-10 minutes), he will take the train to his girlfriend's house. It is clear that if you were to randomly pick a point on the timeline, there would be an 8/10 probability that he would go to the mom's house.

3. Three points are selected randomly on the circumference of a circle. What is the probability that the triangle formed by these three points contains the center of the circle?

We laid out the circumference of a circle as a line segment, of length 1. We called the endpoints both A because they would meet up (be the same point) on the circle.



Then we put 2 points on this line, point B and C. In order to fulfill the parameters of the problem, the distance between any of the points (the lengths of the 3 line segments) has to be less than half of the line segment (0.5).



We know this because, if we think of the line segment as a circle again, if any segment were greater than 0.5, half of the circle would have no points in it, so the center would not be contained within the inscribed triangle. Back with the line segment, we found the inequality for each of the segments, A-B, B-C, C-A.

A —— B —— (0.5) —— C —— A
(0) (0.5) (1)

With the intent of graphing these equations, we set the linear position of B and C to X and Y respectively. A was both 0 and 1, as it was both ends of the line segment.

0 —— X —— (0.5) —— Y —— 1

Our equations were (see Figure 1):

$1-Y<.5$
$Y-X<.5$
$X<.5$

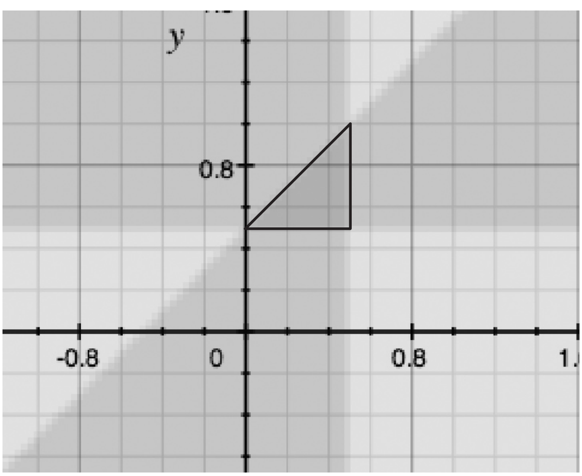
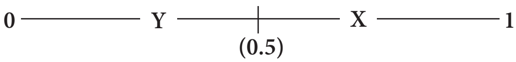


Figure 1

These represent only half the number of events that fulfill the parameters. We made an assumption here that X is greater than Y (or that X=B, Y=C), but the points could be switched, B could be Y and C could be X.



Then we'd have another set of equations with the variables reversed (see Figure 2):

1-X<.5
X-Y<.5
Y<.5

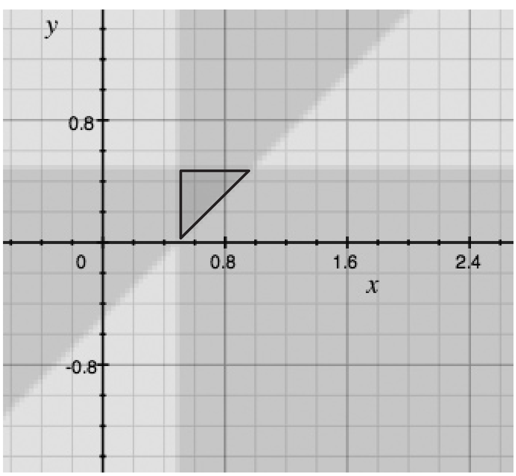


Figure 2

When we graphed these two sets of equations, the parts that overlapped (the solution) formed two congruent triangles.

Together these equations represent the total number of events that fulfill the parameters of the problem. However, to get the probability, we needed the total number of events possible. For this, we created another equation system (see Figure 3):

$X<1$
$Y<1$
$X>0$
$Y>0$

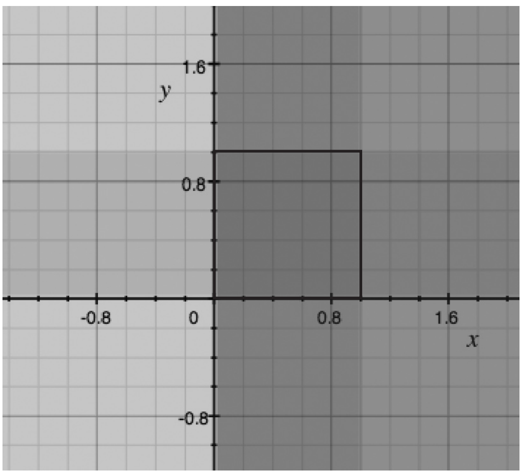


Figure 3

We then graphed this set and got a 1 by 1 square, which contained the previous two congruent triangles. The sum of the area of the two triangles was a quarter of the area of the square. Therefore the probability that the triangle formed by three points contains the center of the circle is ¼. ●