How the Height of the Ramp a Cart is Released From Affects the Stopping Distance of That Cart on Plastic Grass

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1 Abstract

This experiment was designed to determine how the height of a ramp on which a cart is placed affects the stopping distance of that cart. My lab partners, Andrew Cardozo and Meghana Renavikar, and I adjusted the ramp to four different heights, released the cart, and measured the distance past the edge of the ramp that the cart stopped. We found that the higher the ramp, and therefore the greater the potential energy, the greater the distance the cart went before stopping. Although this outcome was along the lines of the accepted data, we found that while the increase between potential energy and stopping distance should have been linear, in our case, as the velocity^2 and potential energy increased, the stopping distance increased by less than a linear relationship would suggest.

2 Hypothesis

We predicted that the higher the height of the ramp, the greater the stopping distance. We also predicted that the relationship between the potential energy of the cart and the stopping distance would be linear. To start, potential energy is calculated by mass*gravity*height, and therefore potential energy is proportional to the height of the ramp. Stopping distance is determined by the amount of kinetic energy an object possesses, which, at the bottom of the assumed frictionless ramp, is equal to the potential energy the cart possessed at the top of the ramp. Stopping distance is calculated as d=W/F. Therefore, as W (the work needed to stop the cart, which is equal to the cart's poten-

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tial energy at the top of ramp) increases, d (stopping distance) should increase by the same amount (a linear relationship) because F, the frictional force, is constant. We predicted that the relationship between velocity and stopping distance of the cart would be quadratic. This is because the equation for an object's kinetic energy is $KE=\frac{1}{2}mv^2$. Therefore, as velocity goes up by x amount, the kinetic energy of that object goes up by x^2 amount. The relationship between kinetic energy and stopping distance is linear, so the relationship between velocity and stopping distance should be quadratic.

3 Experiment Design

3.1 Equipment

"Frictionless" cart
"Frictionless" cart ramp
Meter stick
Scale
Metal Elevator

3.2 Procedure

1. Use the scale to find the mass of the cart.

2. Find a level area of turf, and place the elevator on it.

3. Place the ramp so that one end is on the turf, while the other end of the ramp is resting on the elevator.

4. Lower the elevator to its lowest height.

5. Place the cart on the ramp so its back wheels are at the back edge of the ramp.

6. Measure the distance from the back of the cart down to the ground and the distance from the front of the cart to the ground.

7. Average these two distances and use the average as the cart's height.

8. Release the cart.

9. Measure the distance from the edge of the ramp to the front wheels of the cart.

10. Repeat step 5, 8, and 9 two times.

11. Raise the elevator to a new height and repeat steps 5-9 three times.

12. Repeat step 11 until you have stopping distance data for the cart for at least four different ramp heights.

4 Results

Starting Height (cm)	Potential Energy (J)	Theoretical Velocity at the Bottom of Ramp (m/s)	Actual Stopping Distance (cm)	Average Stopping Force (N)	Calculated Coefficient of Friction on Turf (Based on Average of Average Stopping Forces)
10.15	.4929	1.41	24.4	2.02	0.43
13.85	.6725	1.648	34.7	1.94	
19.25	.9348	1.943	45.2	2.07	
26.45	1.2844	2.278	53.9	2.39	

Table 1: *The stopping distance, theoretical velocity, and stopping forces for a cart on an inclined plane starting at various heights.*

5 Discussion

In order to determine how the height of a cart on an inclined plane affects its stopping distance, we released a cart from the top of a ramp, which was set at four different heights. We measured the cart's initial height by averaging the distance from the ground of the back and the front of the cart, and then after the cart stopped on the turf, we measured the distance from the edge of the ramp to the front wheels of the cart. We found both that as the height of the ramp increased, the stopping distance increased, and that the coefficient of friction for turf is 0.43.

We found this by placing a cart on the top of a ramp and releasing the cart and measuring the distance of the cart from the front wheels. For the measurement of the height of the cart, we measured from the back of the cart to the ground and from the front of the cart to the ground. We then averaged these two measurements, which would give us the height from the middle of the cart. We did this because we had to take into account the fact the cart was on an inclined plane rather than a flat surface. Therefore, we could not simply measure from the front of the cart as that would neglect the fact that the back of the cart has a greater height than the front, and we couldn't measure from the back of the cart as that would neglect the fact that the front of the cart was lower. In order to balance these factors, we took the average of those two measurements. The second measurement we took was from the bottom of the ramp to the front wheels of the cart. We took this measurement from the front wheels because the stopping distance measures how long friction is acting on the cart, turning the cart's kinetic energy into heat and bringing the cart to a stop. As soon as the cart's front wheels come off the ramp they touch the turf. The equation for friction, $f=\mu N$, tells us that as soon as there is force on the ground the frictional force is present. Therefore, we must measure the stopping distance as the length of distance friction is acting on the cart, which is from where the front wheels touch the turf to where the front wheels stop. However, this does present a small problem, because while the back wheels are on the ramp, and the front wheels are on the ground, not all of the force of the cart is on the ground, so the normal force is not equal to the cart's mass*gravity. However, this is only a short distance and so presents a relatively small source of error.

Our results confirmed our overarching hypothesis that as the height, and therefore potential energy, of the cart increased, the stopping distance likewise increased. However, our results show that some of the relationships between velocity, starting height, and stopping distance in this experiment were not entirely in line with our hypothesis. To demonstrate this, let's begin with the relationship between theoretical velocity and starting height. Figure 1 shows that this relationship is of the form $y=\sqrt{x}$, which confirms our hypothesis. The equation for kinetic energy is $KE=\frac{1}{2}mv^2$ so as velocity goes up x amount, the kinetic energy goes up by x^2. This kinetic energy at the bottom of the ramp should theoretically (neglecting friction) be equal to the potential energy of the cart at the top of the ramp. The potential energy at the top of the ramp is calculated as PE=mgh. Therefore, PE is proportional to h, the starting height. Because KE is proportional to v^2, PE=KE, and PE is proportional to h, the relationship of velocity vs. height should be of the form $y=\sqrt{x}$. As the starting height goes up x amount, the velocity goes up amount.

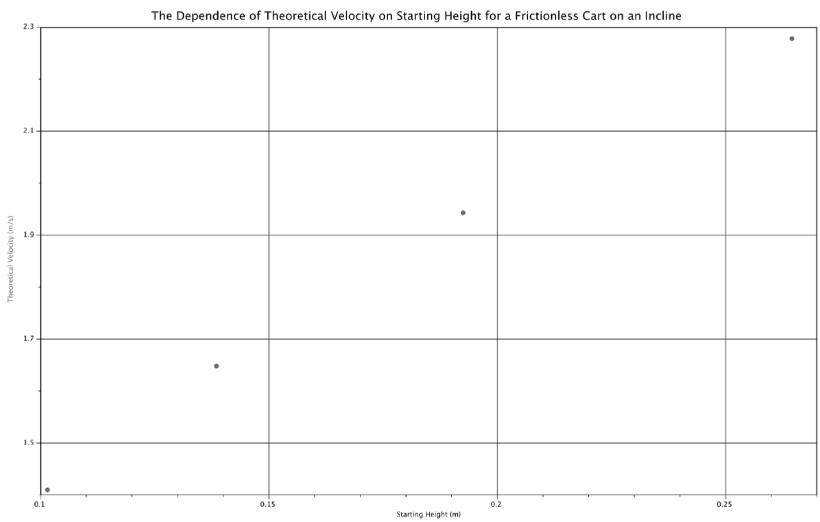

Figure 1: *Dependence of theoretical velocity on starting height for a frictionless cart on an incline.*

Figure 2 shows the relationship of stopping distance vs. potential energy. This graph seems to tell us that the relationship is of the form $y=\sqrt{x}$, which would deny our hypothesis. The graph should show a linear relationship, because if the ramp is frictionless the potential energy at the top equals the cart's kinetic energy at the bottom, so the work done to bring the cart to a stop should be equal to the cart's potential energy at the top. W=Fd, so the work is directly proportional to the stopping distance. Therefore, the relationship between the potential energy and the stopping distance should be linear. This graph could be interpreted as revealing a linear relationship, but it shows that the more potential energy increases, the less the stopping distance changes. This gives it a shape more closely associated with that of a graph of the form $y=\sqrt{x}$. However, this discrepancy can be explained through work and energy. As the height of the ramp increases, so does the angle between the ramp and the ground. Therefore, the greater the starting height, the more vertical the cart hits the ground. As the cart hits the ground, it does work and transfers some energy into the ground. The greater the angle between the ramp and the ground, the more force the cart hits the ground with, and therefore the more energy is lost to the ground, which would explain why the relationship looks less linear than anticipated.

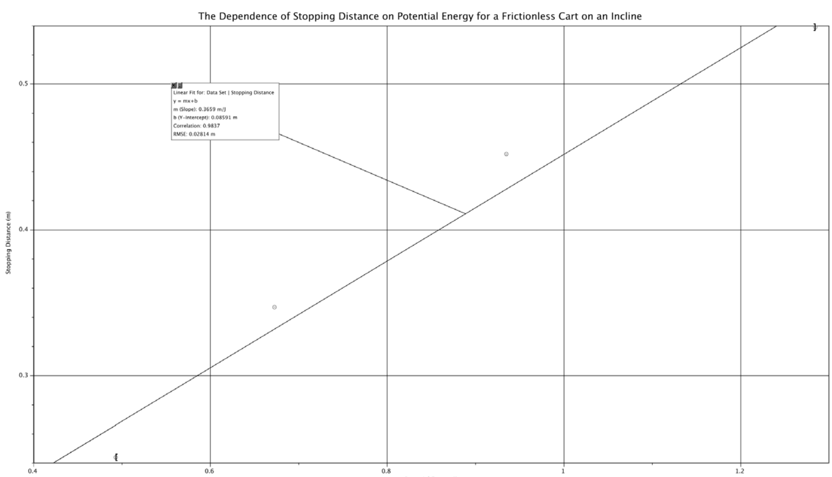

Figure 2: *Dependence of stopping distance on potential energy for a frictionless cart on an incline.*

Figure 3 shows the relationship of stopping distance vs. velocity^2. The graph once again seems to suggest that the relationship is of the form $y=\sqrt{x}$, which would deny our hypothesis. This relationship should be linear. The stopping distance is calculated through a transformation of the equation W=Fd to d=W/F. F is constant, so d is directly proportional to W. W is the work needed to stop the cart, which is equal to the kinetic energy of the cart. Kinetic energy is calculated by $KE=\frac{1}{2}mv^2$. Through transformation, this equation becomes $v=\sqrt{\frac{2KE}{m}}$

KE is equivalent to W, which is equal to Fd.

Therefore $v=\sqrt{\frac{2Fd}{m}}$

If we then square velocity as this graph tells us to, we end up with $v^2=\frac{2Fd}{m}$

This tells us that the relationship between velocity squared and stopping distance should be linear. This graph is vaguely linear, which supports this conclusion to some degree. But it does possess a shape more closely related to $y=\sqrt{x}$, which can be explained the same way as the similar shape in Figure 2. As the starting height increases, the angle between the ramp and the ground increases, leading the cart to exit the ramp and hit the ground with a greater downward force, losing more of its kinetic energy. The starting height is proportional to the cart's potential energy, which is equal to kinetic energy, proportional to velocity^2. This explains why the shape of Figure 2 is slightly linear but also has a $y=\sqrt{x}$ shape, as each increase in velocity^2 results in more kinetic energy but also more energy lost to the ground.

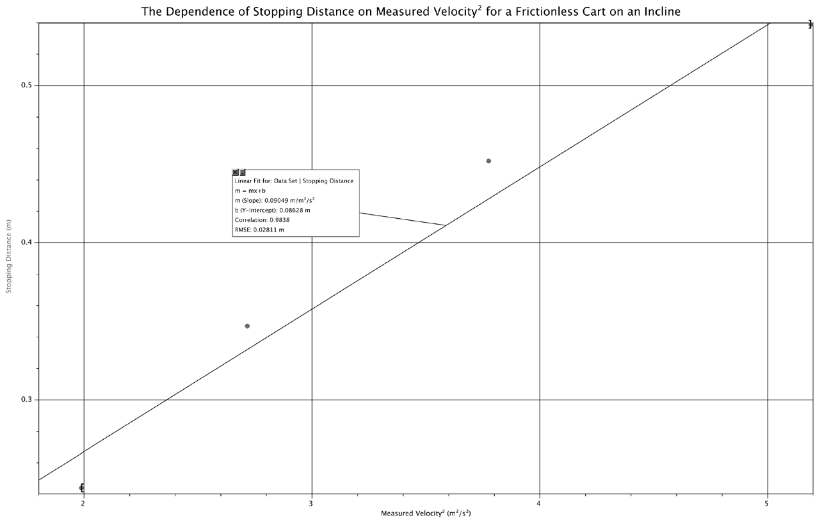

Figure 3: *Dependence of stopping distance on measured velocity² for a frictionless cart on an incline.*

There were two significant sources of error in this experiment. First, we did not take into account friction on the ramp. Although the cart and ramp were designed to be as frictionless as possible, this does not eliminate the presence of friction. Because of this unaccounted-for force, the potential energy at the top and kinetic energy at the bottom for the cart would actually not be the same. This could be improved on by finding the value of the frictional force between the cart and the ramp, multiplying this by the length of the track and subtracting this quantity from the potential energy. This would then give us an accurate assessment of the cart's actual kinetic energy at the bottom of the track. Although air resistance also creates some friction, I do not believe it is significant enough to have greatly affected our results.

The second source of error was already discussed to some degree: the changing angle of the ramp and the ground. This affected our results because the greater the angle there is, the greater amount of the cart's kinetic energy was imparted into the ground when the cart came off the tracks, which affected the stopping distance of the cart. This could be solved by using a longer ramp, and instead of adjusting the height of the elevator, which changes the angle, simply releasing the cart from different points on the ramp, so that the height of the cart changes while the angle of the ramp does not.

These two sources of error can both be used to analyze why the average stopping force is not the same for all four starting heights. In theory, it should be because the stopping force, F, is simply the product of the coefficient of friction, a constant, and the normal force, another constant, since we used the same cart for all of our starting heights. However, the average calculated stopping forces are, in order from lowest starting height to greatest, 2.02 N, 1.94 N, 2.07 N, and 2.39 N. With the exception of the first, the average stopping forces clearly get greater as the starting height increases. This is directly related to the second source of error. The greater the starting height, the more energy was lost in the initial impact with the ground, which led to a shorter stopping distance. Work=Force * distance, so distance decreases due to the lost energy. However, this lost energy is not taken into account on the work side of the equation. Therefore, the force appeared to increase.

If we were to adjust this experiment to vary the mass of the cart rather than the starting height, and then plot a graph of stopping distance versus potential energy, we would see different results. In our original lab, when we varied the height, we changed the potential energy, just as we do when we change the mass of the cart in this new lab since PE=mgh. However, stopping distance (d) is determined by W/F, and in the original lab, we only changed the work (W) needed to bring the car to a halt by increasing the starting height. In this new lab, we change both W and F by adjusting the mass. W changes the same way as it did in the original lab since it is equivalent to the cart's initial potential energy (if we ignore the friction of the track.) Force (F), also increases because F is friction, which is calculated by $f=\mu N$. N=mg, the mass of the object multiplied by gravity. In this case, when we change the mass of the object, we change the normal force, which changes the frictional force the ground applies to the cart, which in turn affects the stopping distance. Therefore, in our original lab, the graph would show that as potential energy increases, so does stopping distance. In this new lab, changing the mass would change the potential energy and the frictional force, and therefore even as potential energy increased, the stopping distance would not. ●